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A General Higgs Sector: Constraints and Phenomenology

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Abstract

We have investigated some phenomenological aspects of an $SU(2)\times U(1)$ scenario where scalars belonging to arbitrary representations of $SU(2)$ are involved in electroweak symmetry breaking. The resulting interaction terms are derived. Some constraints are obtained on the arbitrary scalar sector from the requirement of tree-level unitarity in longitudinal gauge boson scattering. We also show that, in cases where the scalars ensure $\rho = 1$ at tree-level, useful restrictions on their parameter space follow from precision measurements of the $Zb\bar{b}$ vertex. Finally, some salient features about the production of such Higgs bosons in e^+e^- collision are discussed.

1. Introduction

The electroweak symmetry breaking sector of the Standard Model (SM) is still an object of widespread conjectures. Admittedly, the model with one Higgs doublet is the simplest one and is also consistent with the current experimental results. From an alternative standpoint, however, it may seem to be rather artificial to postulate just *one* fundamental scalar doublet in nature, as compared with several families of particles in the fermionic sector. Models with two or more doublets have been explored in this spirit [1].

It is also pertinent to investigate the consequences of scalars belonging to other representations of $SU(2)$. This will not alter the gauge group structure of the SM, but will enlarge its particle content and change the gauge-scalar and fermion-scalar interactions in a significant manner. The motivation for introducing higher Higgs representations can be seen, for example, in the context of neutrino masses. If lepton number can be violated, then the interaction with the vacuum expectation value (VEV) of a Higgs triplet may lead to Majorana masses for left-handed neutrinos [2]. This not only facilitates the incorporation of neutrino masses without any need for completely sterile right-handed neutrinos, but also explains why neutrino masses are so much smaller than their charged fermionic counterparts. Triplet Higgs scenarios have been further explored in the literature in both the contexts of e^+e^- and hadronic colliders [3, 4, 5, 6, 7, 8, 9].

However, representations of scalars higher than doublets suffer from one

serious malady, namely, they in general violate the experimental constraints on the ρ-parameter, defined as $\rho = m_W^2/m_Z^2 \cos^2\theta_W$. The tree-level value of ρ in a general Higgs scenario is given by [1]

$$\rho = \frac{\sum_k [4T_k(T_k+1) - Y_k^2]|v_k|^2 c_k}{\sum_k 2Y_k^2 |v_k|^2} \tag{1}$$

where v_k is the VEV of the k-th multiplet of scalars, and $c_k = 1(1/2)$ for a complex (real) representation. It can be verified from above that higher scalar representations contradict the experimentally measured value of $\rho = 1.0004 \pm 0.0030$ [10] unless one of the following conditions is satisfied:

1. We have some higher Higgs representation which *accidentally* does not contribute to ρ, such as one with $T = 3$, $Y = 4$.

2. The VEVs of the higher representations are much smaller than the doublet VEV so that ρ is not affected significantly.

3. There is some custodial symmetry among the higher representations such that their contributions to ρ cancel each other.

It should be noted that although the last possibility smacks of fine-tuning, especially where higher order effects are taken into account, some serious model-building has been done in recent times on its basis [11, 12, 13].

Based upon the experiences summarised above, we attempt in this paper to discuss the state of affairs in a general scenario with an arbitrary collection of scalar representations, both real and complex. We have also tried to

see if constraints other than those from the ρ-parameter can be imposed on such a scenario. It will be shown below, for example, that the experimental measurement of the effective $Zb\bar{b}$ vertex can lead to restrictions on an arbitrary Higgs structure (if it is not already constrained by the ρ-parameter), which are comparable with, and sometimes more stringent than, those on two-Higgs doublet models. That the longitudinal gauge boson scattering amplitudes should respect unitarity at high energies also lead to some non-trivial constraints on the scalar sector.

In Section 2 we discuss the various interactions in the Lagrangian with a general assortment of Higgs multiplets. The constraints on them arising from the requirement of tree-level unitarity in longitudinal gauge boson scattering are derived in Section 3. In Section 4 we take up the limits from the $Zb\bar{b}$ effective vertex. Some further phenomenological implications, like Higgs production by the Bjorken process and the effects of an $H^{\pm}W^{\mp}Z$ tree-level interaction, are discussed in Section 5. We conclude our discussions in Section 6, and follow up with some of the formulae listed in the Appendices.

2. Formalism

A scalar multiplet in the $SU(2) \times U(1)$ context can occur in both real ($Y = 0$) and complex ($Y \neq 0$) representations. In general, the Lagrangian in the scalar sector contains both the kinetic energy and the potential terms. The potential, of course, has to depend on the particle content of specific

models. Since our purpose here is to investigate the general or model-independent features of an arbitrary Higgs structre, we wish to refrain from attaching ourselves to any particular form of the potential. The general potential consists of all possible gauge-invariant quadratic and quartic terms formed out of all the fields present in the scenario. The imposition of some additional symmetry or phenomenological requirement eliminates or relates some of these terms. It should be noted that any particular choice of the potential determines the scalar self-couplings and the mixing matrices relating the weak eigenstates to the physical (or mass) eigenstates.

The kinetic term of the Higgs fields, which also includes the interaction terms with the electroweak gauge fields by virtue of the covariant derivative, can be written as

$$\mathcal{L}_{kin} = \sum_k [(D_\mu \Phi_k^c)^\dagger (D^\mu \Phi_k^c) + \frac{1}{2}(D_\mu \Phi_k^r)^T (D^\mu \Phi_k^r)], \tag{2}$$

where $\Phi^c(\Phi^r)$ denotes a complex (real) Higgs multiplet, and a sum over repeated Greek indices is implied. The explicit form of the covariant derivative is

$$D_\mu = \partial_\mu + ig\sum_{i=1}^{3} W_\mu^i T_i + \frac{i}{2} g' Y B_\mu, \tag{3}$$

where

$$T^\pm = T_1 \pm iT_2 \tag{4}$$

and T_3 are the $SU(2)$ generators of proper dimension.

Here let us clarify our notations. We will denote the Higgs fields, both complex and real, in the weak basis by ϕ (and the multiplets by Φ) and those

in the mass basis by H. These two sets will be related by unitary matrices. The neutral scalar and pseudoscalar fields do not mix with each other; thus, we have two distinct mixing matrices in the neutral sector. All Higgs fields with same nonzero charge can mix with each other. We assume these mixing elements to be real so that there is no CP violation in the Higgs sector. This does not affect the generality of our treatment since we will focus on CP conserving phenomena. Thus, we can have the following relations between the fields in the weak and the mass basis:

$$\begin{aligned}
H_i^{\pm} &= \alpha_{ij}\phi_j^{\pm}, \quad \phi_i^{\pm} = \alpha_{ji}H_j^{\pm}, \\
H_i^{\pm\pm} &= \sigma_{ij}\phi_j^{\pm\pm}, \quad \phi_i^{\pm\pm} = \sigma_{ji}H_j^{\pm\pm}, \\
H_{Si}^{0} &= \beta_{ij}\phi_{Sj}^{0}, \quad \phi_{Si}^{0} = \beta_{ji}H_{Sj}^{0}, \\
H_{Pi}^{0} &= \gamma_{ij}\phi_{Pj}^{0}, \quad \phi_{Pi}^{0} = \gamma_{ji}H_{Pj}^{0},
\end{aligned} \tag{5}$$

and similarly other relations between the triple- and higher-charged Higgs fields can be written. As we have mentioned before, the specific forms of the mixing matrices α, β etc. depend on the potential. The elements of the first row of the α and γ matrices are fixed from the definition of the Goldstone bosons, which of course include the VEVs of the scalar fields which are in its turn, functions of the potential. We denote the charged Goldstone bosons $G^{\pm}$ as $H_1^{\pm}$, and the neutral Goldstone boson G^0 as H_{P1}^0. The subscripts S and P denote the scalar and the pseudoscalar states respectively.

The Goldstone states can directly be found out from the Lagrangian by

looking at the non-diagonal pieces (which are cancelled by the gauge-fixing term). The expressions for the Goldstones immediately follow:

$$G^+ = \frac{g}{\sqrt{2}m_W}\sum_k [(T^+ v_k)^\dagger \phi_k - \phi_k^\dagger (T^- v_k)], \tag{6}$$

$$G^- = \frac{g}{\sqrt{2}m_W}\sum_k [\phi_k^\dagger (T^+ v_k) - (T^- v_k)^\dagger \phi_k], \tag{7}$$

$$G^0 = \frac{ig}{2\cos\theta_W m_Z}\sum_k [\phi_k^\dagger Y_k v_k - v_k Y_k \phi_k], \tag{8}$$

which are normalized to the SM expressions. The sum here runs over both the real as well as the complex representations. However, G^0 gets contribution only from the complex representations, as $Y_{real} = 0$. Also, one has to introduce a factor of $1/2$ for real representations as both the particle and the antiparticle states are in the same multiplet.

If we make the additional assumption that for all complex representations, $T^- v = 0$, i.e., the neutral member of the multiplet has the lowest weight, then we have $T = Y/2$ for all such representations. Such a choice includes, without any loss of generality, all doublets as well as the triplet models which are of current phenomenological interest [1]. It simplifies the form of the charged Goldstone bosons given above in the sense that $T^- v_k$ terms will be absent:

$$\begin{aligned} G^+ &= \frac{g}{\sqrt{2}m_W}\sum_{k=1}^{n} [\sqrt{n_k - 1} v_k^c + \frac{1}{2}\sqrt{n_k^2 - 1} v_k^r]\phi_k^+ \\ &= \sum_k \alpha_{1k}\phi_k^+. \end{aligned} \tag{9}$$

Here, v_k^c (v_k^r) is the VEV of the k-th complex (real) representation, whose dimension is n_k. Note that one row of the matrix α gets determined in this way.

With $T^- v = 0$, the gauge boson masses are given by

$$m^2_{W^\pm} = \frac{1}{2} g^2 \sum_k [T(T+1)(v^r_k)^2 + 2T(v^c_k)^2] \tag{10}$$

$$m^2_Z = \frac{1}{2} g^2 \sum_k 4T^2 \sec^2\theta_W (v^k_c)^2. \tag{11}$$

Thus, the real multiplets do not contribute to m_Z, whereas both the real as well as the complex multiplets do contribute to m_W. This also recasts eq. (1) into

$$\rho = \frac{\sum_k [T(T+1)(v^r_k)^2 + 2T(v^c_k)^2]}{\sum_k 4T^2 (v^c_k)^2}. \tag{12}$$

Thus, a good way to have the custodial $SU(2)$ symmetry intact with more than one 'bad' representations is the following. We can have one or more doublets or singlets with arbitrary VEVs; they do not affect ρ. Next, we add one complex n-plet ($T = Y/2$) with neutral VEV $= b$, and $\frac{4T-2}{T+1}$ (if integer) number of real n-plets ($T = n,\ Y = 0$) with same VEV. However, if $\frac{4T-2}{T+1}$ is not an integer, one has to add a single real n-plet (for example) with VEV $= (\frac{4T-2}{T+1})^{1/2} b$. (Note that this option is available also when $\frac{4T-2}{T+1}$ is an integer.) For $T = 1$, this prescription reproduces the triplet Higgs model of Georgi and Machacek [12]. For $T = 2$, we have a complex and two real 5-plets with same VEV, which respect a custodial $SU(2)$ symmetry. This will henceforth be called the 5-plet model.

The forms for the vertex factors directly follow from eq. (2). We show some typical vertices which will be directly relevant in our future discussions. Appendix 1 contains a more complete list. Here, we adopt the simplification

that for a complex representation $T^- v_k = 0$.

First, let us show some couplings involving two gauge bosons and one scalar. The $ZZ\phi^0_{Sk}$ vertex factor is given by $\frac{ig^2}{\sqrt{2}\cos^2\theta_W} v_k Y_k^2 g_{\mu\nu}$. (The $ZZ\phi^0_{Pk}$ term is forbidden from parity conservation.) In the mass basis, this factor can be written as

$$ZZH^0_{Si} \ : \ \frac{ig^2 g_{\mu\nu}}{\sqrt{2}\cos^2\theta_W} \sum_k \beta_{ik} v_k Y_k^2 \tag{13}$$

which is obvious from eq. (5). The remaining interactions are given below in the mass basis only. For example,

$$W^+W^-H^0_{Si} \quad : \quad \frac{ig^2 g_{\mu\nu}}{\sqrt{2}} \sum_k \beta_{ik}(v_k^c Y_k + \frac{1}{4}(n_k^2 - 1)v_k^r) \tag{14}$$

$$W^+ZH^-_i \quad : \quad \frac{ig^2 g_{\mu\nu}}{\sqrt{2}\cos\theta_W} \sum_k \alpha_{ik}(f_k^c v_k^c + f_k^r v_k^r) \tag{15}$$

where

$$f_k^c = \sqrt{n_k - 1}(\cos^2\theta_W - Y_k); \quad f_k^r = \frac{1}{2}\sqrt{n_k^2 - 1}\cos^2\theta_W. \tag{16}$$

Another interesting coupling, which occurs with at least real 5-plets or complex triplets, is

$$W^+W^+H^{--}_i \ : \ \frac{ig^2 g_{\mu\nu}}{2} \sum_k \sigma_{ik}(g_k^c v_k^c + g_k^r v_k^r) \tag{17}$$

with

$$g_k^c = \sqrt{2(n_k - 1)(n_k - 2)} \tag{18}$$

$$g_k^r = \frac{1}{4}\sqrt{(n_k^2 - 1)(n_k^2 - 9)}. \tag{19}$$

As emphasized earlier, the expression for g_c^k does depend on the approximation $T^- v_k = 0$; however, g_k^r is independent of any such approximation and uses only the fact that $Y_{real} = 0$.

In the next category for two scalar-one gauge boson couplings, we only show the $ZH_i^+ H_j^-$ vertex:

$$ZH_i^+ H_j^- \;:\; -\frac{ig}{2\cos\theta_W}(p_1+p_2)_\mu[2\cos^2\theta_W\delta_{ij} - \sum_k \alpha_{ik}\alpha_{jk}Y_k] \tag{20}$$

where p_1 is incoming and p_2 is outgoing at the vertex. From this, the expressions for vertices involving one or two charged Goldstones follow immediately:

$$ZH_i^+ G^- \quad : \quad -\frac{ig}{2\cos\theta_W}(p_1+p_2)_\mu[2\cos^2\theta_W\delta_{i1} - \sum_k \alpha_{ik}\alpha_{1k}Y_k], \tag{21}$$

$$ZG^+ G^- \quad : \quad -\frac{ig}{2\cos\theta_W}(p_1+p_2)_\mu \sum_k \alpha_{1k}^2(2\cos^2\theta_W - Y_k). \tag{22}$$

It may be noted that at the proper limit, the couplings corresponding to the SM or the two-Higgs doublet model are successfully reproduced. For a general representation, elements of the mixing matrices depend on the chosen form of the potential. However, we will show that one can extract some model-independent information about them.

From the expressions given above (and also in Appendix 1) one observes that *all scalars* (except a singlet) in general couple to the electroweak gauge bosons. On the other hand, only weak doublets couple with fermions; that too in a restricted manner to avoid unreasonably large flavour-changing neutral currents (FCNC). Following the conditions of natural flavour conservation [14], we will invoke two kinds of models: in the first one, only one doublet

(let us call it Φ_1) couples to both up- and down-type quarks (model 1) and in the second one, Φ_1 couples with the up-type and Φ_2 with the down-type quarks (model 2). Other doublets, if present, do not couple to the fermions [15].

In model 1, the Yukawa couplings are as follows:

$$\bar{u}dH_i^+ \quad : \quad \frac{ig}{\sqrt{2}m_W}\frac{\alpha_{i1}}{\alpha_{11}}(m_uP_L - m_dP_R), \tag{23}$$

$$\bar{u}dG^+ \quad : \quad \frac{ig}{\sqrt{2}m_W}(m_uP_L - m_dP_R). \tag{24}$$

In model 2, their counterparts are

$$\bar{u}dH_i^+ \quad : \quad \frac{ig}{\sqrt{2}m_W}(\frac{\alpha_{i1}}{\alpha_{11}}m_uP_L - \frac{\alpha_{i2}}{\alpha_{12}}m_dP_R), \tag{25}$$

$$\bar{u}dG^+ \quad : \quad \frac{ig}{\sqrt{2}m_W}(m_uP_L - m_dP_R), \tag{26}$$

where

$$P_L = \frac{1-\gamma_5}{2}, \quad P_R = \frac{1+\gamma_5}{2}. \tag{27}$$

Note that no sum over weak eigenstates appears as only one weak doublet is responsible for giving mass to a particular type of quark. The α's in the denominator follow from the normalisation of charged Goldstones. It also follows that for the $t-b$ system, where $m_t \gg m_b$, both models yield the same $\bar{t}bH_i^+$ vertex.

Before concluding this section, we give some specific examples of the scalar mixing matrices. An illustrative case is the triplet model, consisting of a complex ($Y=2$) and a real triplet in addition to the standard doublet,

where the tree-level VEV's of the complex and the real triplet are taken to be equal to maintain $\rho = 1$ [1].

In terms of the doublet-triplet mixing angle θ_H, where $\tan\theta_H = 2\sqrt{2}b/a$ ($a/\sqrt{2}$ and b being the doublet and the triplet VEVs respectively), the mixing matrices α, β and γ have the following form:

$$\alpha = \begin{pmatrix} c_H & s_H/\sqrt{2} & s_H/\sqrt{2} \\ o & 1/\sqrt{2} & -1/\sqrt{2} \\ -s_H & c_H/\sqrt{2} & c_H/\sqrt{2} \end{pmatrix}, \tag{28}$$

$$\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2/3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{3} & \sqrt{2/3} \end{pmatrix}, \tag{29}$$

$$\gamma = \begin{pmatrix} c_H & s_H \\ -s_H & c_H \end{pmatrix}, \tag{30}$$

where

$$\phi^+_{1,2,3} = \phi^+, \chi^+, \xi^+, \tag{31}$$

$$H^+_{1,2,3} = G^+, H_5^+, H_3^+, \tag{32}$$

$$(\phi^0_S)_{1,2,3} = \phi^0_R, \chi^0_R, \xi^0, \tag{33}$$

$$(H^0_S)_{1,2,3} = H^0_1, H'^0_1, H^0_5, \tag{34}$$

$$(\phi^0_P)_{1,2} = \phi^0_I, \chi^0_I, \tag{35}$$

$$(H^0_P)_{1,2} = G^0, H^0_3 \tag{36}$$

[1]It has already been discussed in the literature [3] that maintaining this equality at higher orders requires fine-tuning at a level comparable to that in the minimal SM.

the notations being the same as in Ref. [3], and no mixing between H_1^0 and ${H'}_1^0$ is assumed. This is necessary to fix the first row of the β matrix, as it cannot be fixed from any Goldstone normalisations. From the forms of α and β, it is obvious that $H_5^\pm$, H_5^0 and ${H'}_1^0$ do not couple to fermions. Also, it is easy to check, for example, that the structure of α rules out a $H_3^+W^-Z$ coupling.

Another example is the 5-plet model. It consists of one complex doublet of VEV $a/\sqrt{2}$ and one complex 5-plet and two real 5-plets with VEV= b. As we have already shown, such a choice automatically ensures $\rho = 1$ at tree-level. Defining $\tan\theta_H = 2\sqrt{2}b/a$, we get, for example,

$$\alpha = \begin{pmatrix} c_H & s_H/2 & \sqrt{3/8}s_H & \sqrt{3/8}s_H \\ -s_H & c_H/2 & \sqrt{3/8}c_H & \sqrt{3/8}c_H \\ 0 & -\sqrt{3/4} & 1/\sqrt{8} & 1/\sqrt{8} \\ 0 & 0 & 1/\sqrt{2} & -1\sqrt{2} \end{pmatrix}, \tag{37}$$

where

$$\begin{aligned} \phi^+_{1,2,3,4} &= \phi^+(doublet), \quad \chi_1^+(complex\ 5-plet), \\ &\quad \psi_1^+(real\ 5-plet), \quad \psi_2^+(real\ 5-plet). \end{aligned} \tag{38}$$

It is again obvious that H_3^+ and H_4^+ do not couple to fermion-antifermion pairs.

3. Unitarity Sum Rules

The requirement of unitarity of the partial wave amplitudes for longitudinal gauge boson scattering [16] can lead to some useful restrictions on an arbitrarily extended scalar sector. On one hand, consideration of the zeroth partial wave amplitudes at a centre-of-mass energy much higher than the Higgs masses yields the upper limit of about 1 TeV [17] on *at least* one scalar which interacts with a pair of gauge bosons. On the other hand, one may demand that for arbitrarily large values of the scalar masses, unitarity should hold for $\sqrt{s} \lesssim 1$ TeV (i.e., ask for the same high-energy cut-off as that in the minimal SM). One way of guaranteeing this is to impose the restriction that for each scattering process, the total amplitude for all the Higgs-mediated diagrams be equal to the minimal SM amplitude. We maintain that such conditions are *sufficient* rather than necessary. However, they allow us to relate and simplify the plethora of parameters in a scenario containing an assortment of scalars. Here we present those among such conditions which can be written in a model-independent way, i.e., without recourse to the detailed form of the scalar potential. (For example, computation of scattering processes involving the scalars either in the initial or in the final channel requires scalar self-couplings, and hence are omitted from our set of conditions.) As far as longitudinal gauge boson scattering is concerned, all the relations that follow from the above criterion can be obtained from two processes. We choose $W_L Z_L \to W_L Z_L$ and $W_L^+ W_L^+ \to W_L^+ W_L^+$ for that purpose.

All the other processes can be easily seen to give the same set of conditions using crossing symmetry.

The condition for the amplitude of $W_L Z_L \to W_L Z_L$ channel to satisfy the unitarity bound is

$$g^2\Big[\sum_k v_k^2(Y_k^3 - f_k^2) + (\sum_k \alpha_{1k} f_k v_k)^2\Big] = \frac{1}{2}m_W^2 \tag{39}$$

where the sum is over all multiplets, real and complex, and f_k is the proper factor (f_k^c or f_k^r) as defined in eq. (16). The second term in the left-hand side leaves out the Goldstone contribution. Furthermore, in the left-hand side, the term proportional to Y_k^3 comes from the t-channel graphs whereas the one proportional to f_k^2 comes from the combined s and u-channel graphs; the latter two add with a negative sign to the t-channel diagram to give the total contribution (as at $s,t,u \gg m_H^2,\ s+t+u \approx 0$) which should be equal to the SM contribution. $W_L^+W_L^- \to Z_L Z_L$ gives the same condition from crossing symmetry.

The second condition comes from $W_L^+W_L^+ \to W_L^+W_L^+$, which is related to $W_L^+W_L^- \to W_L^+W_L^-$ by crossing. Here, doubly charged scalars in an extended scenario play a significant role; however, Goldstone contributions need not be subtracted as neutral pseudoscalars do not couple to W-pairs from CP-invariance of the Lagrangian. The condition is

$$\frac{g^2}{2}\sum_k \Big[(Y_k^2 - \frac{1}{2}(g_k^c)^2)(v_k^c)^2 + \{\frac{1}{4}(n_k^2-1) - \frac{1}{2}(g_k^r)^2\}(v_k^r)^2\Big] = m_W^2. \tag{40}$$

Here g_k^c and g_k^r are those given in eqs. (18) and (19). The dependence on the

β- and σ-matrices cancel from completeness.

In processes like $f\bar{f} \to V_L V_L$ $(V = W, Z)$, good high-energy behaviour of the cross-section is ensured by gauge invariance. The same situation holds with a complicated Higgs structure if one demands that the sum of the amplitudes for all scalar-mediated diagrams be equal to the SM Higgs-induced amplitude. However, it is straightforward to see that such an equality is automatically ensured from the unitarity of the β-matrix and the expression for α_{1i} in terms of the scalar VEVs. On the other hand, a similar requirement for the process $f_1\bar{f}_2 \to W_L Z_L$ yields a nontrivial restriction. It involves the $H^{\pm}W^{\mp}Z$ couplings present in a general scenario. A simple calculation gives the following sufficient condition:

$$\frac{f_1^c v_1}{\alpha_{11}} = \sum_k \alpha_{1k}(f_k^c v_k^c + f_k^r v_k^r). \tag{41}$$

If, in addition, the doubly charged scalars have $\Delta L = 2$ couplings with a pair of leptons, then a constraint on the doubly charged sector can be obtained from processes like $e^- e^- \to W_L^- W_L^-$ [8].

4. Constraints from $Z \to b\bar{b}$

Since the advent of microvertex detectors, the decay $Z \to b\bar{b}$ [18] can be tracked down with increasingly higher precision, and the latest results from LEP-1 seriously encourage the view that there may be new physics beyond the SM. The reason is that the experimentally measured value of the ratio

R_b, defined as

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons}}, \tag{42}$$

comes out to be nearly 2.2σ above the SM value; the experiments give 0.2202 ± 0.0020 while the SM prediction is 0.2156 ± 0.0004 [19], for $m_t = 175$ GeV. With the top quark mass more or less accurately known, R_b can act as a 'new physics meter', as most of the QCD corrections to the individual decay widths cancel in the ratio [20], and the new physics can make significant one-loop contribution to the $Zb\bar{b}$ vertex. These contributions are of two types: self-energy corrections to the external b-quark, and vertex corrections in the form of triangle diagrams. However, they are significant only if they involve a t-quark in the loop.

A 2.2σ deviation does not prove the existence of new physics, and there are some doubts about the experimental number; e.g., the experiments may have taken into account $b\bar{b}$ pairs generated from a gluon radiated off a light quark, in which case actual R_b will be smaller. Another value quoted for R_b is 0.2192 ± 0.0018 [21], in which case the deviation is only 2σ. In any case, the result encourages those models which predict a positive deviation, e.g., a large parameter space in the Minimal Supersymmetric Standard Model (MSSM), or models with extra gauge bosons etc. On the other hand, it puts very tight constraint on those models which predict the deviation to be negative — examples are the two-Higgs doublet model [22], and, as will be shown, the triplet model of Georgi and Machacek [12]. But first we will show

our results for a general Higgs sector.

A very concise formulation will essentially follow the notation set by Boulware and Finnell [22]. Let the deviation of R_b from its SM value be denoted by δR_b. Thus, we have

$$\delta R_b = 0.1718\Delta \tag{43}$$

where the factor Δ contains the non-oblique one-loop effects — the oblique parts are already known to have negligible contribution. The charged Higgs coupling to left-handed b-quarks is proportional to m_t while that to right-handed b-quarks is proportional to m_b. Thus, the production of left-handed b-quarks is strongly favoured. Since the same is true for the tree-level case, it will not cause any significant change to the electroweak asymmetries.

In the limit $m_b \to 0$, the effects of new physics can be introduced through a change in the vertex factors for the $Zb\bar{b}$ coupling:

$$\begin{aligned} {v_L^b}' &= v_L^b + \frac{g^2}{16\pi^2} F_L(p^2, m_t), & (44)\\ {v_R^b}' &= v_R^b + \frac{g^2}{16\pi^2} F_R(p^2, m_t), & (45)\end{aligned}$$

where p is the four-momentum of the Z boson. Also, the right- and left-handed couplings of b (and t) quarks with Z at the tree-level are given by

$$\begin{aligned} v_R^b &= \frac{1}{3}\sin^2\theta_W & (46)\\ v_L^b &= -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W & (47)\\ v_R^t &= -\frac{2}{3}\sin^2\theta_W & (48)\\ v_L^t &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W. & (49)\end{aligned}$$

The form factors F_L and F_R can be written as

$$F_{L,R} = \sum_{i=1}^{4} F^i_{L,R} \tag{50}$$

where the individual form factors F^1, F^2, F^3 and F^4 receive contributions from the set of diagrams in Fig. 1(a), (b), (c) and (d) respectively.

Both for models 1 and 2 discussed in Section 2, the i-th charged Higgs couples with b_L with the strength

$$\lambda^i_L = \frac{g}{\sqrt{2}m_W} m_t \frac{\alpha_{i1}}{\alpha_{11}} \tag{51}$$

whereas for the right-handed b, the expression is model-dependent:

$$\lambda^i_R = -\frac{g}{\sqrt{2}m_W} m_b \frac{\alpha_{i1}}{\alpha_{11}} \quad \text{(model 1)} \tag{52}$$

$$\lambda^i_R = -\frac{g}{\sqrt{2}m_W} m_b \frac{\alpha_{i2}}{\alpha_{12}} \quad \text{(model 2).} \tag{53}$$

For model 1, $\lambda^i_R \ll \lambda^i_L$, and the same is true for model 2 unless $\alpha_{i2}/\alpha_{12} \gg \alpha_{i1}/\alpha_{11}$. Thus, even at the one-loop level, the produced b's are dominantly left-handed and the electroweak asymmetries are little affected. Also, it is a relatively safe assumption to neglect the change in v^b_R, which is roughly proportional to $(\lambda^i_R)^2$; the error introduced is of the order of $(m_b/m_t)^2$, or 0.1%.

So $F^i_R \approx 0$, and F^i_Ls are given by

$$F^1_L = \sum_{i=2}^{n} B_1(m_t, m_{H_i^+}) v^b_L [\lambda^i_L]^2, \tag{54}$$

$$F^2_L = \sum_{i=2}^{n} \Big[[-m_Z^2 (C_{11} + C_{23})(m_t, m_{H_i^+}, m_t) - \frac{1}{2} + 2C_{24}(m_t, m_{H_i^+}, m_t)] v^t_R$$

$$
\begin{aligned}
& +m_t^2 C_0(m_t, m_{H_i^+}, m_t) v_L^t \Big] [\lambda_L^i]^2, \qquad (55)\\
F_L^3 \;=\; & \sum_{i,j,k=1}^{n} \Big[-2C_{24}(m_{H_i^+}, m_t, m_{H_j^+}) \Big] \alpha_{ik}\alpha_{jk} (\cos^2\theta_W - \frac{1}{2}Y_k) \lambda_L^i \lambda_L^j \\
& + \sum_{k=1}^{n} 2C_{24}(m_W, m_t, m_W)(\cos^2\theta_W - \frac{1}{2}Y_k)(\alpha_{1k})^2 [\lambda_L^1]^2, \qquad (56)\\
F_L^4 \;=\; & \sum_{i=2}^{n}\sum_{k=1}^{n} -\frac{1}{2}\cos\theta_W \lambda_L^i \alpha_{ik} (f_k^c v_k^c + f_k^r v_k^r) \\
& \times [C_0(m_t, m_W, m_{H_i^+}) + C_0(m_t, m_{H_i^+}, m_W)], \qquad (57)
\end{aligned}
$$

where B and C's are the well-known two- and three-point functions first introduced by Passarino and Veltman [23], and f_k^c and f_k^r are defined in eq. (16). The loop amplitudes are evaluated with dimensional regularisation and $\overline{MS}$ subtraction scheme, and the sum, F_L, is free of all divergences.

The absolute sign of F_L^1 and F_L^2 are straightforward: the first one is negative while the second one is positive. F_L^3 and F_L^4 require a more careful treatment.

What we are planning to do is to place a lower bound on the mass of the charged Higgs(es). For that, the most profitable scheme is to take all physical charged Higgses to be degenerate (or nearly degenerate) in mass. Otherwise, the C_{24} function will be dominated by the lowest mass eigenstate; the bound may be slightly weakened at the cost of moving all other charged scalars to the heavy mass regime. As the mass spectra cannot be investigated without complete specification of the potential, this is the best that one can achieve in a general treatment. This allows us to take the C_{24} function to be nearly

a constant, and using the unitarity of the α matrix, one can recast F_L^3 as

$$
\begin{aligned}
F_L^3 &= \frac{g^2}{m_W^2}\Big[-\frac{1}{(\alpha_{11})^2} C_{24}(m_{H^+}, m_t, m_{H^+})(\cos^2\theta_W - \frac{1}{2}) \\
&\quad + C_{24}(m_W, m_t, m_W) \sum_{k=1}^{n} (\alpha_{1k})^2 (\cos^2\theta_W - \frac{Y_k}{2})\Big]. \qquad (58)
\end{aligned}
$$

The C_{24} function is negative, so the first term on the right-hand side of eq. (58) is positive. The second term is positive for $Y_k \leq 1$, and negative for $Y_k > 1$ (we assume only integer Y for scalar multiplets). Thus, for complex doublets and real non-doublets, F_L^3 is positive definite.

However, here one must note a point. The constraints from $Z \to b\bar{b}$ are meaningful only if ρ is forced to unity at tree-level. Otherwise, the non-doublet VEVs are required to be so much smaller than the doublet VEVs (from the experimental constraint on ρ) that the doublet-nondoublet mixing angle θ_H is very small, and $Z \to b\bar{b}$ does not put any significant constraint on m_{H^+}.

The tree-level value of ρ can be forced to unity in models with doublet and/or singlet scalar multiplets, or in models where the effects of 'bad' representations cancel out. In these models, all α_{1k}'s are specified from the definition of the charged Goldstone. If m_{H^+} is of the same order of m_W, both the C_{24} functions on the right-hand side of eq. (58) are approximately same. In this case, F_L^3 *always* turns out to be positive and proportional to $\tan^2\theta_H$. Even for a larger m_{H^+}, F_L^3 is always positive. Also, these models necessarily imply $F_L^4 = 0$. These two important results are proved in Ap-

pendix 2. Thus, $F_L = F_L^1 + F_L^2 + F_L^3$, which turns out to be positive for $m_{H^+} \leq 1$ TeV. Δ in eq. (43) is given by

$$\Delta = \frac{g^2}{16\pi^2} \frac{2v_L^b}{(v_L^b)^2 + (v_R^b)^2} F_L, \tag{59}$$

and thus δR_b is negative, thus tightly constraining the parameter space for these models. The lower bound on m_{H^+} is inversely proportional to $\tan^2\theta_H$; if $\alpha_{11} = \cos\theta_H = 1$, all other α_{1k}'s are zero, and F_L is also zero; thus there is no bound on m_{H^+}. It is also evident that the approximate magnitude of the bound is independent of the number of 'bad' representations.

A word about F_L^4, though it may be nonzero only in some contrived models (e.g., one with a $T = 3, Y = 4$ multiplet). Using the same logic as before, we can take C_0 to be a constant, and F_L^4 can approximately be written as

$$\begin{aligned} F_L^4 &= -\frac{g m_t}{2\sqrt{2} m_W} \cos\theta_W C_0(m_t, m_W, m_{H^+}) \Big[(\cos\theta_W - \sec\theta_W) v \\ &\quad - \frac{1}{v^2} \sum_{k=1}^{n} \{ (v_k^c)^2 f_k^c + (v_k^r)^2 f_k^r \} \Big] \end{aligned} \tag{60}$$

where $m_W^2 = g^2 v^2/2$. C_0 being positive, the first term in the RHS is positive and gives negative δR_b; the second term adds in the same direction if $Y_k \leq 1$.

The above discussion reflects a rather interesting complementarity. Scenarios which do not ensure $\rho = 1$ at tree-level cannot be restricted unequivocally using R_b; however, such scenarios are severely constrained by the value of ρ itself. If, on the other hand, the tree-level value of ρ is somehow fixed at unity through some additional symmetry in a complicated scalar sector,

then these very conditions which ensure $\rho = 1$ make it possible to always constrain the scalar sector through R_b. Thus the near-unity of ρ, coupled with precision measurement of R_b, limits the parameter space of an arbitrarily extended Higgs structure.

As a nontrivial example of a model with custodial symmetry preserving scalar sector, we again consider the triplet model. In this model, $F_L^4 = 0$ and the rest F_L^i's are all proportional to $(\lambda_L^3)^2$. Thus, only H_3^+ can be constrained — H_5^+ does not couple with the fermions and do not contribute to the loop-amplitude. λ_L^3 being proportional to $\tan\theta_H$, we take two representative values: $\sin\theta_H = 0.5$ and $\sin\theta_H = 0.8$. m_t is varied over the range 176 ± 13 GeV. The resulting δR_b's are shown in Figures 2 and 3 respectively.

For $\sin\theta_H = 0.5$, the lower bound on $m_{H_3^+}$ is too small to be of any significance. Even direct experiments put a better bound. For models with $\rho_{tree} \neq 1$, $\sin\theta_H$ is even smaller. For $\sin\theta_H > 0.8$, the lower bound on $m_{H_3^+}$ is almost 1 TeV. Thus we can say that $\sin\theta_H = 0.8$ is the maximum mixing allowed in this model, unless one wants to have a scalar with mass above 1 TeV. Note that this is about $3-4$ times stronger than the bound derived by Gunion *et al* [3] from FCNC processes, and also relatively free from hadronic uncertainties.

5. Higgs Production

The production of non-standard Higgs bosons in both hadronic and e^+e^-

colliders has been extensively discussed in the literature. In particular, the production of scalars with exotic charges (like H^{++}) has received considerable attention [4, 7]. In this section, we include a brief discussion on the production of non-standard scalars belonging to arbitrary multiplets in e^+e^- collisions.

A. Neutral Scalar Production

The production of a neutral scalar that can couple to a pair of gauge bosons can always take place through the Bjorken process $e^+e^- \rightarrow Z \rightarrow Z^* H^0_{Si}$ in a Z-factory (or $e^+e^- \rightarrow Z^* \rightarrow Z H^0_{Si}$ in a higher energy e^+e^- machine) [1]. In the general case, the scalars produced in this manner will decay into a pair of fermions (say, $b\bar{b}$), and the final state will consist of four fermions, two of which will have an invariant mass equal to the respective scalar mass. However, many extended models (aimed at keeping $\rho_{tree} = 1$) possess some additional symmetries that forbid the interaction of some of these scalars with fermions. In such cases, those scalars will decay either into four fermions, induced by a pair of real or virtual gauge bosons, or into two fermions via loops. Assuming that the former mode dominates, the ratio of the contributions to $Z \rightarrow 4f$ and $Z \rightarrow 6f$ channels from all the scalar mass eigenstates is given by

$$\frac{\Gamma(Z \rightarrow \sum_{i=1}^{m} Z^* H^0_{Si} \rightarrow 4f)}{\Gamma(Z \rightarrow \sum_{i=m+1}^{n} Z^* H^0_{Si} \rightarrow 6f)} = \frac{\sum_{i=1}^{m} \sum_{k,l} G_i \beta_{ik} \beta_{il} v_k^2 v_l^2 Y_k Y_l}{\sum_{i=m+1}^{m} \sum_{k,l} G_i \beta_{ik} \beta_{il} v_k^2 v_l^2 Y_k Y_l}, \tag{61}$$

where it has been assumed only that the physical scalar states, from $i = 1$

to m, couple to fermions. G_i is given by

$$G_i = \int_{x_0}^{x_1} g(x)\ dx, \tag{62}$$

with

$$g(x) = \frac{(1 - x + \frac{x^2}{12} + \frac{2}{3}y^2)(x^2 - 4y^2)^{1/2}}{(x - y^2)^2} \tag{63}$$

where

$$y = m_{H^0_{Si}}/m_Z. \tag{64}$$

It is clear from above that the maximum value (with all degenerate scalars) of the four-fermion signals via Bjorken process is that for the single Higgs doublet in the SM. Also, the neutral components of real scalar multiplets do not contribute to any of the two kinds of signals. The above result can be extended in a straightforward way to $e^+e^- \to Z^* \to \sum_i ZH^0_{Si}$ at a higher energy.

B. Neutral Pseudoscalar Production

Pseudoscalars are produced through the mechanism $Z(Z^*) \to H^0_{Si}H^0_{Pj}$. The process and the ensuing signals are closely analogous to the signal of the pseudoscalar in two-Higgs doublet models [24]. The only interesting difference might occur if some pseudoscalars do not have tree-level couplings to fermions. In such cases, the pseudoscalar decays through channels involving real or virtual gauge bosons or scalars.

C. Charged Scalar Production

While the mode $Z(Z^*) \to H_i^+ H_j^-$ is still open, a new avenue for charged scalar production opens up when higher Higgs representations are present. The $H_i^\pm W^\mp Z$ vertex in a general scenario leads to the process $Z(Z^*) \to H_i^\pm W^{*\mp}(W^\mp)$. The width for $Z \to H_i^\pm W^{*\mp} \to H_i^\pm(p_3) f_1(p_1) \bar{f}_2(p_2)$ is given by

$$\Gamma_i = \mathcal{G}_i \sum_{k,l} t_k t_l \alpha_{ik} \alpha_{il} \tag{65}$$

where

$$t_k = (\cos\theta_W - Y_k \sec\theta_W)\sqrt{n_k - 1} v_k \tag{66}$$

and

$$\mathcal{G}_i = \frac{1}{8\pi^2 m_Z} \int dE_1 dE_2 |\mathcal{M}|^2, \tag{67}$$

$|\mathcal{M}|^2$ being the relevant squared matrix element. the signal of a charged Higgs produced in this way will consist chiefly of two- or four-fermion decay modes, depending upon whether the $H_i^\pm$ couples to fermions or not at the tree-level. Again, the latter possibility is often the consequence of symmetries imposed on the scalar sector to maintain $\rho_{tree} = 1$. Thus a real or virtual Z in e^+e^- machines will give rise to four- or six-fermion signals as a consequence of charged Higgs production through the $H_i^\pm W^\mp Z$ vertex. Again, assuming that $(n - m)$ out of n singly charged scalars do not have tree-level fermionic coupling, one obtains

$$\frac{\Gamma(Z \to 4f)}{\Gamma(Z \to 6f)} = \frac{\sum_{i=2}^{m} \sum_{k,l} \mathcal{G}_i \alpha_{ik} \alpha_{il} t_k t_l}{\sum_{i=m+1}^{n} \sum_{k,l} \mathcal{G}_i \alpha_{ik} \alpha_{il} t_k t_l}, \tag{68}$$

where $i = 1$ has been left out of the sum in order to separate out the charged Goldstone field. In the formula above, any $H_i^\pm$ will have $H_i^\pm f_1 \bar{f}_2$ interaction if

(i) $\alpha_{i1} \neq 0$ in the case where only the doublet Φ_1 gives masses to all the fermions, and

(ii) $\alpha_{i1}, \alpha_{i2} \neq 0$ in the case when Φ_1 and Φ_2 are responsible for the masses of up- and down-type fermions respectively.

The observable consequences of the $H_i^\pm W^\mp Z$ vertex in both LEP-1 and higher energy machines have been discussed in detail in the context of the triplet model [5, 6]. It has also been shown [5] that in cases where the $H_i^\pm$ does not couple to fermions, its dominant decay mode is the tree-level one into four fermions over most of the parameter space.

We illustrate in Fig. 4 the branching ratio for $Z \to H_i^\pm \ell \bar{\nu}_l$ as a function of the $H_i^\pm$ mass in the triplet model. It is clear from the graphs that a considerable range of the parameter space of m_{H^+} and θ_H, the doublet-triplet mixing angle, can be constrained from the existing experiments.

6. Conclusions

We have discussed the phenomenology of a general scenario with an arbitrary combination of real and complex Higgs multiplets in arbitrary representations of $SU(2)$. We have seen that with some very modest assumptions, most of the interactions and formulae in such a scenario can be obtained

in rather simplified and physically transparent form. When the multiplets are such that they do not ensure $\rho = 1$ at the tree-level, the experimental value of ρ itself is the most stringent constraint on them. On the other hand, for those models where $\rho_{tree} = 1$ is ensured by suitable contrivance, the precision measurement of $\Gamma(Z \to b\bar{b})$ strongly constrains the parameter space. The consideration of unitarity sum rules in longitudinal gauge boson scattering can also yield interesting relationships among the parameters in a general structure.

On the whole, an extended scalar structure in the electroweak symmetry breaking scheme has a rich phenomenology and deserves unbiased scrutiny. The search for such 'exotic' Higgs particles should therefore be given due priority in all the present and future experiments.

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Appendix 1

In this appendix, we present the vertex factors for a general assortment of scalar multiplets. We display only those vertex factors which do not depend explicitly on the form of the scalar potential. Thus, the three-scalar and four-scalar vertices are omitted.

1. Two gauge bosons and one scalar:

$$ZZH^0_{Si} \quad : \quad \frac{ig^2}{\sqrt{2}\cos\theta_W} g_{\mu\nu} \sum_k \beta_{ik} v_k Y_k^2, \tag{1.1}$$

$$W^+W^-H^0_{Si} \quad : \quad \frac{ig^2}{\sqrt{2}} g_{\mu\nu} \sum_k \beta_{ik} (Y_k v_k^c + \frac{1}{4}(n_k^2 - 1) v_k^r), \tag{1.2}$$

$$W^+ZH^-_i \quad : \quad \frac{ig^2}{\sqrt{2}\cos\theta_W} g_{\mu\nu} \sum_k \alpha_{ik} (f_k^c v_k^c + f_k^r v_k^r), \tag{1.3}$$

$$W^+W^+H^{--}_i \quad : \quad \frac{ig^2}{2} g_{\mu\nu} \sum_k \sigma_{ik} (g_k^c v_k^c + g_k^r v_k^r), \tag{1.4}$$

where f_k^c, f_k^r, g_k^c and g_k^r are defined in eqs. (16), (18) and (19).

2. Two scalars and one gauge boson:

(p_1 and p_2 are respectively incoming and outgoing momenta at the vertex)

$$ZH_i^+H_j^- \quad : \quad -\frac{ig}{2\cos\theta_W}(p_1+p_2)_\mu [2\cos^2\theta_W \delta_{ij} - \sum_k \alpha_{ik}\alpha_{jk} Y_k], \tag{1.5}$$

$$ZH^0_{Si}H^0_{Pj} \quad : \quad \frac{g}{2\cos\theta_W}(p_1+p_2)_\mu \sum_k \beta_{ik}\gamma_{jk} Y_k, \tag{1.6}$$

$$ZH_i^{++}H_j^{--} \quad : \quad -\frac{ig}{2\cos\theta_W}(p_1+p_2)_\mu [4\cos^2\theta_W \delta_{ij} - \sum_k \sigma_{ik}\sigma_{jk} Y_k], \tag{1.7}$$

$$W^-H_i^+H^0_{Sj} \quad : \quad -\frac{ig}{\sqrt{2}}(p_1+p_2)_\mu \sum_k \alpha_{ik}\beta_{jk} (h_k^c + h_k^r), \tag{1.8}$$

$$W^- H_i^+ H_{Pj}^0 \quad : \quad \frac{g}{\sqrt{2}}(p_1+p_2)_\mu \sum_k \alpha_{ik}\gamma_{jk}(h_k^c + h_k^r), \tag{1.9}$$

$$W^- H_i^{++} H_j^- \quad : \quad -\frac{ig}{\sqrt{2}}(p_1+p_2)_\mu \sum_k \sigma_{ik}\alpha_{jk}(q_k^c + q_k^r), \tag{1.10}$$

where

$$h_k^c = \sqrt{n_k - 1} \quad ; \quad h_k^r = \frac{1}{2}\sqrt{n_k^2 - 1}; \tag{1.11}$$

$$q_k^c = \sqrt{2(n_k - 2)} \quad ; \quad q_k^r = \frac{1}{2}\sqrt{n_k^2 - 9}. \tag{1.12}$$

In general, in the weak basis, if W couples with two members of charges j and $j+1$ in a particular multiplet of dimension n, then the vertex factor is given by

$$W^- \phi^{+(j+1)}\phi^{-j} \quad : \quad -\frac{ig}{\sqrt{2}}(p_1+p_2)_\mu \sqrt{(n-j-1)(j+1)} \;\; \text{(complex)}, \tag{1.13}$$

$$W^- \phi^{+(j+1)}\phi^{-j} \quad : \quad -\frac{ig}{2\sqrt{2}}(p_1+p_2)_\mu \sqrt{n^2 - (2j+1)^2} \;\; \text{(real)}, \tag{1.14}$$

which can be easily translated to the mass basis if the mixing matrices are known.

3. Two gauge bosons and two scalars:

These are presented below in the weak basis.

$$W^+ W^- \phi^{+Q}\phi^{-Q} \quad : \quad -ig^2 g_{\mu\nu}[T(T+1) - (Q - \frac{Y}{2})^2], \tag{1.15}$$

$$W^+ W^+ \phi^{-(Q+2)}\phi^{Q} \quad : \quad -i\frac{g^2}{2} g_{\mu\nu}(T^+)^2, \tag{1.16}$$

$$W^+ W^+ \phi^{--}\phi^{0} \quad : \quad -i\frac{g^2}{2} g_{\mu\nu} g_k^c \;\; (g_k^r \text{ for real}), \tag{1.17}$$

$$
\begin{aligned}
AA\phi^Q\phi^{-Q} &: \quad -ig_{\mu\nu}e^2Q^2, && (1.18)\\
ZZ\phi^Q\phi^{-Q} &: \quad -ig_{\mu\nu}\frac{g^2}{4\cos^2\theta_W}(2Q\cos^2\theta_W - Y)^2, && (1.19)\\
AZ\phi^Q\phi^{-Q} &: \quad -ig_{\mu\nu}\frac{eQg}{\cos\theta_W}(2Q\cos^2\theta_W - Y), && (1.20)\\
AW^-\phi^{Q+1}\phi^{-Q} &: \quad -ig_{\mu\nu}\frac{eg(2Q+1)}{\sqrt{2}}T^+, && (1.21)\\
ZW^-\phi^{Q+1}\phi^{-Q} &: \quad -ig_{\mu\nu}\frac{g^2}{\sqrt{2}}\ [\cos\theta_W(2Q+1) + Y\frac{\cos 2\theta_W}{\cos\theta_W}]\ T^+, && (1.22)
\end{aligned}
$$

where T^+ is a shorthand notation for $< \phi^{Q+1}|T^+|\phi^Q >$.

Appendix 2

Here we will state and prove two theorems about the vertex correction to the process $Z \to b\bar{b}$, in which singly charged scalars of arbitrary representation of $SU(2)$ take part.

Definition: By $\rho_{tree} = 1$ models, we mean those models whose scalar sector consists of (i) either complex doublets (and singlets), which guarantee $\rho = 1$ at tree-level, or (ii) a set of 'bad' representations whose effects on $\rho - 1$ at tree-level cancels out, and which have been constrained according to our prescription laid down in Section 2, apart from the usual doublet. There may be more than one doublet, and singlets too. Moreover, we confine ourselves to those complex multiplets for which $T^- v_k = 0$.

Theorem 1: F_L^3, as defined in eq. (58), is always positive for $\rho_{tree} = 1$ models.

Proof: The proof will be given for case (ii) of the above definition only, as the proof for case (i) follows trivially.

Let us take the VEV of the doublet Φ_1, which gives mass to the top quark, to be v_d. We also consider one complex m-plet, Φ_2, with VEV$=v_m$ $(T = Y/2)$, and one real m-plet, Φ_3, with VEV$=$ $v'_m = \sqrt{4T - 2/T + 1} v_m$.

The Goldstone boson is defined as

$$G^+ = \frac{g}{\sqrt{2} m_W} \sum_i [(T^+ v_i)^\dagger \phi_i^+] = \sum_i \alpha_{1i} \phi_i^+, \tag{2.1}$$

as we take $T^- v = 0$, without any loss of generality. Also,

$$\begin{aligned} T^+ v &= \sqrt{2T} v \quad \text{(complex)}, \\ &= \sqrt{T(T+1)} v \quad \text{(real)}. \end{aligned} \tag{2.2}$$

Thus,

$$(\alpha_{11})^2 = \frac{g^2}{2m_W^2} v_d^2, \tag{2.3}$$

$$(\alpha_{12})^2 = \frac{g^2}{2m_W^2} 2T v_m^2, \tag{2.4}$$

$$(\alpha_{13})^2 = \frac{g^2}{2m_W^2} T(4T-2) v_m^2. \tag{2.5}$$

Therefore,

$$\begin{aligned} \sum_k (\alpha_{1k})^2 (\cos^2\theta_W - \frac{1}{2} Y_k) &= \frac{g^2}{2m_W^2} (v_d^2 + 4T^2 v_m^2)(\cos^2\theta_W - \frac{1}{2}) \\ &= \cos^2\theta_W - \frac{1}{2}, \end{aligned} \tag{2.6}$$

as $g^2(v_d^2 + 4T^2 v_m^2) = 2m_W^2$.

C_{24} being always negative for the arguments used in the definition of F_L^3, and $\cos^2\theta_W > 1/2$ and $\alpha_{11}^2 \leq 1$, it follows that F_L^3 is positive definite. The same proof follows for F_R^3, and also for models with more than one set of 'bad' representations and/or more than one doublets.

Theorem 2: $F_L^4 = 0$ for $\rho_{tree} = 1$ models.

Proof: We take the same assortment of scalars. F_L^4 is proportional to

$$\sum_{i=2}^{n} \sum_{k=1}^{n} \frac{\alpha_{i1}}{\alpha_{11}} \alpha_{ik} (f_k^c v_k^c + f_k^r v_k^r), \tag{2.7}$$

which can be rewritten as

$$\frac{1}{\alpha_{11}} f_k^1 v_d - \sum_{k=1}^{n} \alpha_{1k} (f_k^c v_k^c + f_k^r v_k^r). \tag{2.8}$$

Putting the values of f_k^c, f_k^r, v_k^c and v_k^r, as shown in Theorem 1, we find that the expression *vanishes identically.*

In the proof of the above two theorems, we have assumed that m_{H^+} is of the same order of m_W. The theorems are valid for $m_{H^+} \sim 1$ TeV, beyond which the perturbative unitarity breaks down.

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Figure Captions

1. Feynman diagrams involving charged scalars contributing to the one-loop correction to the $Z \to b\bar{b}$ vertex.

2. δR_b plotted against the charged Higgs mass for $\sin\theta_H = 0.5$. The solid, dotted and dashed lines correspond to $m_t = 163, 176, 189$ GeV respectively.

3. Same as in Figure 2, but with $\sin\theta_H = 0.8$.

4. Branching ratio for $Z \to H^+ \ell \bar{\nu}_l$ plotted against m_{H^+}. The solid and dashed lines correspond to $\sin\theta_H = 0.1$ and 0.8 respectively.